

# Prudential Saving: Evidence from a Laboratory Experiment

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This Version: June 19, 2011

## Abstract

Prudence is a behavioral attitude that is broadly applicable to settings involving risk. It has particular importance in intertemporal choice theory, where it can be interpreted as the intensity of intertemporal substitution. Prior laboratory experiments to elicit prudence have addressed it in a pure-risk sense, by examining behavior in static lotteries and other gambles. It is tempting to impute these results into an intertemporal context, leveraging the fact that “risk aversion” and “elasticity of intertemporal substitution” are directly mappable under expected utility. However, many empirical studies of intertemporal behavior suggest that the two ideas may be distinct. To address prudence in its intertemporal sense, we instead design a small-scale laboratory experiment around a two-period consumption/savings model in order to detect prudence via savings choices. The utility concept in this model disentangles risk preferences from intertemporal preferences, and suggests the type of exogenous variation to present to subjects in the experiment. The experimental design involves a “multiple price list” with scenarios involving income risk and interest-rate risk. In each scenario, subjects must choose how much of their first-period income to save for the second period. The design also implements field-like wealth levels and real time lags to ameliorate the possibility of the decisions being a laboratory artifact. We estimate risk and intertemporal preferences at the individual level using a subject’s savings data and the model’s structural Euler equation. Excluding outliers, the average coefficient of relative risk aversion is 2.06, the average elasticity of intertemporal substitution is 0.75, and the average coefficient of relative prudence is 3.90. These averages mask a good deal of subject-level heterogeneity, as the respective coefficients of variation are, at a minimum, 70%.

**Keywords:** Prudence, precautionary savings, Kreps-Porteus, laboratory experiment, time delay

**JEL classification:** D90, C91, C81

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# 1 Overview

Kimball (1990) introduced the term “prudence” to describe the effect of a positive third derivative of a utility function on a risky decision. Intuitively, prudence measures the sensitivity of a choice to risk. To quantify this sensitivity, Kimball derived coefficients of absolute and relative prudence that are analogous to the Arrow-Pratt coefficients of risk aversion. The prudence coefficients involve derivatives of the marginal utility function (*i.e.*,  $u'''$  and  $u''$ ) instead of derivatives of the utility function. While the notion of risk aversion (as the propensity to avoid risky situations altogether) is empirically well-established, the extent to which people are prudent is currently an open question. In this paper, we approach this question by collecting savings data in a laboratory context and estimating a structural Euler equation with these data.

Our approach requires us to simultaneously assess prudence in the risk and time domains. The theoretical literature on prudence largely studies its effect on each domain independently. The intertemporal line of research stresses the importance of a positive third derivative for capturing reasonable precautionary-savings motives (*cf.* Drèze and Modigliani 1972, Leland 1968, Sandmo 1970). Indeed, precautionary savings is a paradigmatic example of prudence discussed by Kimball. The static-risk line of research emphasizes the equivalence between a positive third derivative and aversion to downside risks (*cf.* Menezes et al. 1980). There has been little examination of whether “coefficients of prudence” are transferrable between these domains.<sup>1</sup> The transferrability question is of the essence when constructing an empirical strategy for estimating prudence when both intertemporal and risk components are present.

The literature on risk aversion contains many indications that “coefficients of risk aversion” are not transferrable between the static-risk context (*e.g.*, decisions involving gambling and insurance) and time (*e.g.*, decisions involving savings and long-term investment), even though the elasticity of intertemporal substitution can be mathematically represented as the inverse of relative risk aversion in many cases. For example, an oft-cited macroeconomic study by Hall (1988) finds elasticities of intertemporal substitution near 0, which are generally incompatible with estimates

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<sup>1</sup>Kimball’s derivation of prudence relies on the general insight that the convexity of marginal utility with respect to a random variable is decisive for an individual’s optimal choice of a control variable (Rothschild and Stiglitz 1971). Because no assumptions are made about the context of the choice problem, the coefficients of prudence by themselves carry no empirical context.

of risk aversion.<sup>2</sup> A similarly classic laboratory experiment involving static risk by Holt and Laury (2002) finds increasing relative risk aversion, which would generate odd predictions if applied to many intertemporal-choice problems.<sup>3</sup>

It is likely that any discrepancies in risk aversion will also translate into discrepancies in prudence, since both make use of the marginal utility function. To address this issue, we operationalize the preference specification of Kreps and Porteus (1978) and Selden (1978) (hereafter SKP) in our analysis. SKP provide a choice-theoretic framework that separates risk and intertemporal attitudes. Kimball and Weil (2009) (hereafter KW) re-evaluate the coefficients of prudence with SKP preferences. KW show that choice sensitivity to intertemporal risks can arise from both the risk domain and the intertemporal domain, making SKP prudence substantially more complex than a third derivative in either dimension. KW prudence instead involves an interaction between risk aversion, risk tolerance, and the elasticity of intertemporal substitution.

Epstein and Zin (1989, 1991) and Weil (1990) operationalize SKP preferences in an empirical macroeconomic context by restricting the risk domain to constant relative risk aversion and the intertemporal domain to constant elasticity of intertemporal substitution. To our knowledge, no empirical analysis has allowed additional flexibility in each domain's attitudes, or measured prudence in the joint intertemporal and risk contexts at an individual level. In this paper, we present a new experimental design and associated econometric strategy to investigate KW prudence within individual subjects. Our method has three components:

1. Our theoretical starting point is a two-period consumption/savings model with SKP preferences used in lieu of expected utility. The theoretical interaction between risk, time, and SKP preferences allows us to investigate potential savings responses to variations in the exogenous variables (*i.e.*, risks, time lags, and income levels). We use these results to construct a set of scenarios that could potentially identify prudence in a structural estimation of the

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<sup>2</sup>Hall's methodology has been critiqued on various theoretical, econometric, and data grounds. Other studies have found evidence for positive elasticity (*cf.* Patterson and Pesaran 1992, Beaudry and van Wincoop 1996, Biederman and Goenner 2008).

<sup>3</sup>If an individual exhibits increasing relative risk aversion in an intertemporal problem, his or her implicit elasticity of intertemporal substitution approaches 0 as the intertemporal utility-generating quantity (*e.g.*, consumption) increases. In such a case, the intertemporal consumption ratio would not change with increasing wealth, a feature somewhat difficult to reconcile with empirical evidence (see the previous footnote).

model, assuming the savings responses are sufficiently consistent with the model.

2. The experimental design implements these scenarios, which involve risk and a real time lag. The presence of both risk and intertemporal elements activates both components of subjects' decision-making.
3. Using each subject's savings data, we estimate the structural Euler equation associated with the model. We use a flexible yet parsimonious formulation of SKP preferences, which allows subjects to have constant, increasing, or decreasing risk aversion, as well as constant, increasing, or decreasing elasticity of intertemporal substitution.

This “structural-experimental” approach to analyzing savings behavior has a few advantages. First, our behavioral tasks are grounded in a theoretical context that has been quite extensively studied. Second, we can choose experimental scenarios that give us some reasonable *a priori* expectation of obtaining prudent behavior from subjects, if it exists. In other words, we can choose exogenous variation that will be helpful in identifying our behavior of interest. Third, because the savings data are so completely rooted in the theoretical model, we can use the model and data to estimate SKP preferences. With subject-level preferences in hand, we can then classify individuals along their risk and intertemporal attitudes.

This approach does involve one important tradeoff: we must assume the existence of KW prudence from the start. Concerns about the appropriateness of this assumption are not entirely unwarranted; for example, in a laboratory experiment involving only the risk domain, Deck and Schlesinger (2010) find that some subjects exhibit imprudence. However, the underlying consumption/savings model is not well-behaved under imprudence – precautionary savings motives disappear. Indeed, much of the theoretical literature on intertemporal choice notes exactly this point, showing prudence to be a prerequisite to many stylized empirical facts (*e.g.*, the existence of consumption smoothing). As a result, we are inclined to let the existence of imprudence remain an open empirical question, and operate under the assumption that prudence exists. Indeed, if our approach fails due to the presence of imprudence, it will be immediately obvious as a failure to identify reasonable estimates of the preference parameters.

A critical element in this process is the experimental design that provides our savings data. Because we are trying to separately identify risk and intertemporal preferences, it is important to activate both components of the decision mechanism. To address the intertemporal component, we conduct the experiment in two stages, corresponding to the two periods in the theoretical model. In the first stage, subjects are presented with a list of scenarios involving a first-stage income, a second-stage income, and a second-stage interest rate. Either the second-stage income or interest rate of each scenario is risky. Subjects can save some of the first-stage income in each scenario, and any savings earn interest in the second stage. The savings amounts are open-ended, the only restriction being satisfaction of the budget constraint. After a subject enters savings amounts for all scenarios, two are randomly chosen for payment. Subjects are immediately paid the first-stage income less savings for their two scenarios. After a real time lag of at least a few days, subjects return for the second stage, in which the risky elements of the two scenarios are revealed. The second-stage payment, which involves the second-stage income plus savings with interest from both scenarios, is made immediately thereafter.

We address the risk component by altering the riskiness of the second-stage income and interest rate. Also, to ameliorate a Rabin (2000)-style calibration critique, we scale the income levels quite broadly, from the tens to hundreds of USD. Indeed, Holt and Laury (2002) show that risk attitudes can change substantially with the scale of gambles, so it is potentially quite important to generate data from both low and high scalings in order to get a reasonable calibration of preferences in the risk domain.

Because we have relatively little *a priori* knowledge of how well the parameters in the risk and intertemporal domains are identified from savings data, we conduct four small sessions of our experiment with a variety of time lags and wealth scalings. Because of this variation, our experiment should probably be viewed as a pilot or small-sample study of prudence attitudes. But even with these limited data, we can provide some general typings of risk and intertemporal characteristics, as a proof-of-concept of the analytical strategy. Indeed, we find that subjects tend to be quite heterogeneous in their risk attitudes.

The estimation step of our analysis revealed a rather surprising empirical regularity: a subject's

non-laboratory background consumption level appears to be essential for understanding his or her savings decisions in the experiment. Failing to incorporate any background consumption causes the model to fit very poorly for every subject. Fortunately, our theoretical framework is robust to the inclusion of background consumption, so this result does not immediately invalidate our choice of scenarios. It does, however, indicate that economic forces external to the laboratory setting affected subjects' decisions within the laboratory.

In our opinion, this spillover is probably caused by an interplay between our subject pool and the payoff scale. Subjects were undergraduate students for whom a few hundred USD spread over a few weeks represented a significant increase in consumption. (The financial aid office of the associated university estimates that an average undergraduate spends about USD 1600 per month in consumption-like expenditures.) Hence, our experiment provided a relatively large, unexpected shock to their consumption possibilities set, one that likely brought a real consumption-smoothing element into the laboratory savings decisions. The fact that the experimental decisions were not “compartmentalized” within the laboratory is not particularly troubling, because these savings decisions more closely reflect the behavior of ultimate interest. It does, however, mean that we need to be careful about surveying subjects extensively about their consumption behavior. This is a design feature that we did not originally anticipate, and our ability to address it *ex post* is limited. We provide some robustness checks to verify the degree of bias.

The remainder of this paper is structured as follows. In Section 2, we discuss some related empirical and experimental research. In Section 3, we present the consumption/savings model, and show how prudential predictions are affected by introducing SKP preferences. We describe the experimental design and econometric results in Section 4, and comment on their implications for our understanding of prudence in Section 6. Instructions for the experiment can be found in Appendix A.

## 2 Related Literature

Many intuitive theoretical results in a variety of fields in economics rely on the presence of prudence. These include support for the concavity of the consumption function (Carroll and Kimball 1996,

2001), the effect of background risk on mitigating risk-taking behavior (Eeckhoudt et al. 1996, Gollier and Pratt 1996), the relationship of optimal taxation and insurance under precautionary labor supply (Anderberg and Andersson 2003, Low and Maldoom 2004, Netzer and Scheuer 2007), strategic behavior in uncertain environments such as auctions, pollution problems, and rent-seeking games (Esö and White 2004, Bramoullé and Treich 2009, Treich 2010), the hedging demand for assets under return predictability (Gollier 2008), and the term structure of interest rates (Gollier 2010). Welfare analysis in intertemporal settings also relies on prudence: decreasing relative risk aversion – arising when relative prudence is sufficiently stronger than relative risk aversion – causes the “social” discount rate to decline over time (Gollier 2002a,b),<sup>4</sup> giving relatively higher weight to longer-term outcomes over shorter-term ones.

In contrast to these theoretical riches, the empirical evidence for prudence is remarkably inconclusive. Dynan (1993) uses a second-order Taylor approximation of an Euler equation to estimate relative prudence from US consumption data, and finds a negligible coefficient of relative prudence.<sup>5</sup> Using varied methodologies and datasets, higher but mutually inconsistent estimates have been found (Kuehlwein 1991, Merrigan and Normandin 1996, Eisenhauer 2000, Ventura and Eisenhauer 2006). Bostian and Heinzl (2010) provide parametric structural estimates of prudence coefficients using a flexible utility specification similar to the one in this paper. Estimating a dynamic stochastic general equilibrium model on US macroeconomic data, they find evidence for decreasing relative risk aversion and decreasing relative prudence, although the magnitudes of the declines are small.

Laboratory experiments evaluating prudence have all used its risk definition, *i.e.*, along the lines of Menezes et al. (1980) and Eeckhoudt and Schlesinger (2006).<sup>6</sup> In a variant of the Holt and Laury (2002) lottery-choice design, Tarazona-Gomez (2007) presents subjects with a list of binary

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<sup>4</sup>The social discount rate is equivalent to the inverse of the expected stochastic discount factor, a ubiquitous element of intertemporal decision-making.

<sup>5</sup>Under power utility, the coefficient of relative prudence is  $-u'''(x)x/u''(x) = 1 + \rho$ , where  $\rho$  is the coefficient of relative risk aversion and the inverse of the elasticity of intertemporal substitution. With this specification, Dynan estimates relative prudence to be between 0.14 and 0.16, a range inconsistent with most structural macroeconomic estimates of  $\rho$  (usually between 1 and 4, *cf.* Meyer and Meyer 2005). Lee and Sawada (2007) re-estimate Dynan’s model under the assumption of liquidity constraints and find relative prudence to be between 0.838 and 1.094, a substantially higher estimate that is still incompatible with empirical estimates. These conflicting results illustrate that econometric tests of prudence can potentially be susceptible to specification error.

<sup>6</sup>Eeckhoudt and Schlesinger very generally define prudence as a preference for associating two untoward events – a sure loss of wealth and an additional independent zero-mean risk – to two different random events, instead of to just one of them. This definition does not rely on any particular choice-theoretic framework, but does correspond to Menezes et al.’s aversion to downside risk in the expected-utility context.

choices between a lottery and a certain payoff. A subject's decisions generate variation in the revealed certainty equivalents, allowing coefficients of risk aversion and prudence to be identified. Deck and Schlesinger (2010) also use lottery choices to test the "risk apportionment" predictions of Eeckhoudt and Schlesinger. In each task, subjects face a lottery with two equiprobable outcomes and must allocate their endowment between a sure gain and a lottery. Because the Eeckhoudt and Schlesinger predictions are minimally restrictive on preferences, their associated findings are primarily qualitative.

Ebert and Wiesen (2011) criticize both of these studies for testing preferences that are more restrictive than those specified by the risk-apportionment theory. In the Tarazona-Gomez design, the coefficients of prudence are derived from "truncated expected utility," in the sense that they come from a third-order Taylor approximation of the utility function. Because Tarazona-Gomez only compares lotteries with equal means and variances, a test for prudence in that design collapses to a test for skewness-seeking. By construction of their apportionment tasks, Deck and Schlesinger also test only for skewness-seeking. Ebert and Wiesen instead compare decisions in lotteries that vary only skewness to decisions in lotteries that implement a broader styling of risks. They conclude that evaluating the prevalence of skewness-seeking is not sufficient to draw comprehensive conclusions about prudence.

It is important to note that none of these experiments implement a savings context or any other intertemporal aspect in their designs. Because risk and intertemporal attitudes may operate separately, this lack of an intertemporal context can potentially confound the translation of their results into intertemporal choice.

A few laboratory studies on intertemporal choice have used real time delays in conjunction with tasks involving risk. In an artefactual field experiment, Andersen et al. (2008) show that simultaneously eliciting risk and time preferences from the same subjects generates more coherent estimates than distinct elicitation. The authors focus on two subject-level parameters: the coefficient of relative risk aversion and the utility discount rate. In the design, data on risk attitudes and time preferences are collected successively via two different "multiple price lists." The first list involves tasks that involve some risk and are immediately rewarded, and the second involves

tasks that involve no risk and are rewarded after some months. With an appeal to the dual-selves model of choice (*cf.* Benhabib and Bisin 2005, Fudenberg and Levine 2006), the authors argue that the responses to first set of tasks are probably temptation-driven, while the responses to latter set are probably self-controlled. If so, then two different behaviors (risk aversion and discounting) are revealed in the responses to each list.

Like Andersen et al., our design also emphasizes the importance of simultaneously collecting data on risk and intertemporal attitudes from the same subject, but we do so without recourse to dual-selves. Instead, we present subjects with one “multiple price list” of scenarios, each containing future risk and a time delay. Our strategy is to sample a subject’s “savings function” in several different locations via these scenarios, and then to estimate his or her savings function using the choice-theoretic structure underlying them. This process yields joint estimates of the subject’s risk and intertemporal preferences. And, by judiciously choosing the variation in scenarios, we can potentially recover the higher-order risk and time preferences implicit in these estimates.<sup>7</sup>

A recent study by Andreoni and Sprenger (2009b), also unrelated to prudence, calls for additional caution when analyzing choices made in an intertemporal context. In their “convex time budgets” design (Andreoni and Sprenger 2009a), subjects are asked to allocate a budget of experimental tokens to sooner and later payments. The two payments are either both certain, both uncertain, or a mixture of certain and uncertain. While an expected-utility model fits the cases where payments are uniformly certain or uncertain, the mixed case reveals a strong preference for certainty. A certainty preference violates the continuity-in-probability assumption between certain and uncertain outcomes of expected utility, and is also at odds with alternative models (*e.g.*, probability weighting under prospect theory). The authors conclude that a distinction is to be made between utility over certain consumption and utility over uncertain consumption. As shown below, an SKP decision-maker first weights an uncertain outcome by its certainty equivalent, and then assigns utility to the certainty-equivalent ranking. This could address the preference for certainty found by Andreoni and Sprenger, because cases without risk (*i.e.*, in which the certainty equivalent is the outcome itself) will tend to be ranked higher.

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<sup>7</sup>From an econometric point of view, the experiment allows us to control exogenous variation in the scenarios that potentially heighten our ability to identify these higher-order attitudes.

### 3 The Two-Period Consumption/Savings Model

Consider a two-period consumption/savings model in which an agent receives an exogenous endowment income in each period.<sup>8</sup> The first period's income can be split between consumption in the first period and savings for the second period. Any amount saved earns a return in the second period. In the von Neumann-Morgenstern (vNM) framework, this model is frequently defined in terms of an additively-separable utility objective and two resource constraints:

$$U(c_1, \tilde{c}_2) = u(c_1) + \beta E_1[v(\tilde{c}_2)] \quad (1)$$

$$y_1 = c_1 + s_1 \quad (2)$$

$$\tilde{y}_2 + s_1(1 + \tilde{r}) = \tilde{c}_2 \quad (3)$$

In this formulation,  $u$  represents the agent's first-period felicity function,  $v$  the second-period felicity function,  $y_t$  the period- $t$  income,  $c_t$  the period- $t$  consumption, and  $s_1$  the first-period savings. From the perspective of the first period, the second period's felicity is discounted by the factor  $\beta$ . Risk can enter from the second-period endowment  $\tilde{y}_2$  and the return  $\tilde{r}$ . We denote a scenario  $\theta = (y_1, \tilde{y}_2, \tilde{r})$  as a parametrization of the exogenous elements of the agent's problem, which includes the applicable probability densities.

Maximizing  $U$  with respect to the savings amount  $s_1$  yields the familiar Euler condition<sup>9</sup>

$$E_1 \left[ \beta \frac{v'(\tilde{c}_2)}{u'(c_1)} (1 + \tilde{r}) \right] = 1. \quad (4)$$

Equation (4) implies that, in equilibrium, the expected discounted net return to savings must be 0%. If this return were greater than 0%, the agent could achieve higher total expected utility by consuming relatively more in the second period (*i.e.*, by saving relatively more in the first period). The converse holds if this return were less than 0%. For each possible state of the world, the agent

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<sup>8</sup>This is the canonical model to study individual precautionary saving (*e.g.*, Drèze and Modigliani 1972, Leland 1968, Sandmo 1970, Kimball 1990).

<sup>9</sup>For expository ease, we assume interior solutions throughout. The model can be easily extended to accommodate decisions at a boundary.

applies the discount

$$\beta \frac{v'(\tilde{c}_2)}{u'(c_1)}$$

to the future return, a quantity that depends on his or her preferences. This stochastic discount factor determines the agent's savings response to any combination of current values and future risks, and thus represents the behavioral kernel of the model.

If the agent's preferences are unknown, one strategy for empirically reconstructing them is to calibrate equation (4) to some savings data. This would involve estimating  $\beta$ ,  $u'$ , and  $v'$  using the variation in the agent's  $s_1$  choices that arises from variations in the scenarios  $\theta$ . Our approach to evaluating individual preferences is in exactly this vein, relying on an incentivized laboratory experiment to generate the scenarios exogenously. As noted previously, however, this expected-utility framework isomorphizes risk and intertemporal preferences. If these are not identical in the agent, simply calibrating equation (4) to savings data cannot provide a reliable indicator of preferences in either domain.

To address this issue, we use the extension of this model by Kimball and Weil (2009) with SKP preferences as a guide for designing our experiment and associated econometric strategy. The utility objective in this case is

$$U(c_1, \tilde{c}_2) = u(c_1) + \beta v(\psi^{-1}(E_1[\psi(\tilde{c}_2)])) \quad (5)$$

where  $\psi$  is a vNM utility function, but  $u$  and  $v$  are not.<sup>10</sup> In this revised model, risk attitudes serve as means of ranking consumption paths by their risk characteristics. Here, the ranking is provided by the certainty equivalent  $\psi^{-1}(E_1[\psi(\tilde{c}_2)])$  of future consumption. Conditional on a risk ranking, the functions  $u$  and  $v$  isolate the preference for consumption now versus later.

The increased flexibility of SKP preferences comes at the cost of a potentially ill-behaved model,

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<sup>10</sup>Rossman and Selden (1978) show that the set of axioms sufficient for the existence of (a) the ordinal functions capturing time preference over the two periods, and (b) a complete set of second-period vNM functions for all possible first-period choices do not imply the existence of a two-period vNM utility function. The authors provide several examples of risk and intertemporal preferences that cannot be represented by two-period vNM utility but are captured by Selden's (1978) "ordinal certainty-equivalent" preferences. Kreps and Porteus' (1978) axiomatization of recursive preferences is equivalent to Selden's in the two-period case. They show that, contrary to intertemporal vNM utility, their framework can capture a preference for the timing of the resolution of uncertainty.

even if the functions  $u$ ,  $v$ , and  $\psi$  are each concave (Gollier 2001). The concavity of  $U$  in  $s$  now also requires the concavity of  $\psi^{-1}(s)$ , and this occurs if the absolute risk tolerance of  $\psi$  is concave or if the functions  $u$  and  $v$  are linear. Otherwise, the problem's first-order condition may yield a local minimum. Our preference specification will ultimately obviate this issue, so for expository purposes, we assume that  $u$ ,  $v$  and  $\psi$  are strictly increasing and strictly concave, and that absolute risk tolerance is strictly concave.

Under SKP preferences, the Euler equation involves a more complex interaction between  $u$ ,  $v$ , and  $\psi$ . Writing it in a form akin to equation (4) yields

$$E_1 \left[ \beta \frac{v'(\psi^{-1}(E_1[\psi(\tilde{c}_2)])) \cdot \psi^{-1'}(E_1[\psi(\tilde{c}_2)]) \cdot \psi'(\tilde{c}_2)}{u'(c_1)} (1 + \tilde{r}) \right] = 1. \quad (6)$$

The new stochastic discount factor contains effects arising from both the risk and intertemporal domains. The first, represented by  $v'(\cdot)$ , reflects the marginal change in future utility from a marginal increase in the certainty-equivalent rank. The second, reflected in  $\psi^{-1'}(\cdot) \psi'(\cdot)$ , reflects the marginal change in certainty-equivalent rank provided by a marginal change in savings. It is worth noting that  $v'(\cdot)$  and  $\psi^{-1'}(\cdot)$  are scalar-valued and not random variables, and so these scale the stochastic discount factor by a constant multiple regardless of the realization of the random elements. However,  $\psi'(\cdot)$  remains a stochastic quantity.

A more intuitive view of equation (4) can be found by grouping the intertemporal and risk functions separately (*cf.* Rossman and Selden 1978):

$$\frac{u'(c_1)}{\beta v'(\psi^{-1}(E_1[\psi(\tilde{c}_2)]))} = E_1 \left[ \frac{\psi'(\tilde{c}_2)}{\psi'(\psi^{-1}(E_1[\psi(\tilde{c}_2)]))} (1 + \tilde{r}) \right]$$

The left-hand side is the agent's marginal rate of substitution between current and future consumption. The right-hand side is a standard asset-pricing equation, with the pricing kernel determined by the agent's risk attitudes. The behavior of the kernel is primarily driven by the stochastic quantity  $\psi'(\tilde{c}_2)$ ; the denominator is scalar-valued and scales the pricing kernel by a uniform amount regardless of the realization of the random elements. The agent thus settles upon an optimal savings amount  $s_1^*$  by introspectively equating the marginal rate of intertemporal substitution with

the opportunity cost of savings (*i.e.*, the risk-adjusted return to savings).

The sensitivity of  $s_1$  to risk can be measured via the precautionary premium, a quantity which reflects the certain compensation needed to counteract the addition of risk to the marginal utility of saving. For small risks with mean zero and vanishing variance, Kimball and Weil use the precautionary premium to derive the following local coefficients of absolute and relative prudence for SKP preferences:

$$AP(c) = ARA_\psi(c) \left( 1 + \frac{\varepsilon_\psi(c)}{RRIS_v(c)} \right) \quad (7)$$

$$RP(c) = RRA_\psi(c) \left( 1 + \frac{\varepsilon_\psi(c)}{RRIS_v(c)} \right) \quad (8)$$

where  $ARA_\psi(c)$  and  $RRA_\psi(c)$  are the Arrow-Pratt coefficients of absolute and relative risk aversion associated with  $\psi$ ,  $\varepsilon_\psi(c) = RP_\psi(c) - RRA_\psi(c)$  is the elasticity of absolute risk tolerance associated with  $\psi$ , and  $RRIS_v(c)$  is the relative resistance to intertemporal substitution associated with  $v$ .<sup>11</sup> KW prudence is thus defined by a mixture of the derivatives of  $\psi$  up to degree 3, and the derivatives of  $v$  up to degree 2. Positive KW coefficients are sufficient for a positive precautionary premium, implying that

$$\varepsilon_\psi(c) > -RRIS_v(c) \quad \Leftrightarrow \quad -\frac{t\psi'''(c)}{\psi''(c)} > RRA_\psi(c) - RRIS_v(c) \quad (9)$$

for local precautionary saving to occur. Note that if  $v$  and  $\psi$  exactly coincide, the original vNM form of  $U$  arises, and equation (9) then reduces to the familiar condition  $\psi'''(c) > 0$ .

Kimball and Weil additionally examine global conditions for prudence under SKP preferences. A globally necessary and sufficient condition is that  $\psi$  exhibits decreasing absolute risk aversion (Gollier 2001), which implies that  $\varepsilon_\psi(c) > 0$  over the entire support. A globally sufficient condition is that  $\psi'$  is convex and  $v$  is more concave than  $\psi$ .

To operationalize these preferences, we choose functional forms for  $u$ ,  $v$ , and  $\psi$ . The nature of  $\psi$  as a risk-preference function suggests the use of a form that can capture a variety of risk attitudes.

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<sup>11</sup>  $RRIS_v(c)$  has the same mathematical form as the coefficient of relative risk aversion of  $v$ . But, to avoid confusing the behavioral effects from the risk and intertemporal domains, we never use “risk” terminology in reference to  $v$ .

Holt and Laury (2002) show that the expo-power function of Saha (1993) can rationalize lottery-choice decisions in an expected-utility framework over a wide range of payoffs, a feature that is essential in our context. The expo-power function has the form

$$\psi(c) = \frac{1}{\alpha_\psi} \left[ 1 - \exp \left( -\frac{\alpha_\psi}{1 - \rho_\psi} c^{1-\rho_\psi} \right) \right] \quad (10)$$

and nests many common risk attitudes: constant absolute risk aversion when  $\rho_\psi = 0$ , constant relative risk aversion as  $\alpha_\psi \rightarrow 0$ , increasing (decreasing) relative risk aversion for  $\rho_\psi < (>) 1$  and  $\alpha_\psi > 0$ , and risk neutrality when  $\rho_\psi = 0$  and  $\alpha_\psi \rightarrow 0$ .<sup>12</sup> Importantly, the degree of risk aversion can vary with  $c$ , potentially ameliorating somewhat the Rabin (2000) critique over a larger range of payoffs than might be possible with a more restricted form. The nature of  $v$  as an intertemporal-preference function suggests the use of a form that can capture a variety of intertemporal substitution patterns. We also use an expo-power function in this case, again because it allows the elasticity of intertemporal substitution to vary with payoffs:

$$v(c) = \frac{1}{\alpha_v} \left[ 1 - \exp \left( -\frac{\alpha_v}{1 - \rho_v} c^{1-\rho_v} \right) \right] \quad (11)$$

It is easily verified that expo-power function has derivatives with alternating sign beginning with positive first, has concave absolute risk tolerance, and exhibits decreasing absolute risk aversion. Hence, the consumption/savings problem is well-behaved under this specification. In addition, except for the case  $\rho_\psi = 0$ , this specification provides a necessary and sufficient condition for KW prudence to exist.

The felicity function  $u$  does not appear in the KW coefficients of prudence. However, because we will estimate  $v$  and  $\psi$  using an Euler equation that involves  $u$ , a good specification of  $u$  is still necessary to avoid bias in these other estimates. The most restrictive option is to assign a simple form to  $u$ , such as linear. While this may be mathematically convenient, the form may be difficult to

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<sup>12</sup>The Arrow-Pratt coefficients of absolute and relative risk aversion of expo-power utility are, respectively,  $ARA(c) = \alpha c^{-\rho} + \rho c^{-1}$ , and  $RRA(c) = \alpha c^{1-\rho} + \rho$ . The combination of  $\alpha \neq 0$  and  $\rho = 1$  yields a power utility function that involves  $\alpha$  instead of  $\rho$ . We do not use this specification to represent power utility, because it reflects a mathematical edge case that can be alternatively represented by first setting  $\rho = 1 + \alpha$  and then letting  $\alpha \rightarrow 0$ .

rationalize. The least-restrictive option is to estimate  $u$  separately, perhaps via another expo-power function. However, this diminishes parsimony and ignores any potential relationship between  $u$  and  $v$ . Indeed,  $u$  and  $v$  both measure felicity over non-stochastic consumption measures. Even though these felicities occur in two different periods, it is plausible that they are evaluated similarly. (This is certainly a common assumption in dynamic macroeconomic models with additively-separable utility.) Hence, we set  $u = v$ , but also provide some evidence on the viability of a separate estimate of  $u$ .

Having fully specified the primitives of the Euler equation, we can, in principle, estimate its preference parameters at the individual level using equation (6). We can then use the estimates of  $v$  and  $\psi$  to generate the KW coefficients of prudence. Grouping elements in equation (6) by time period yields

$$\beta v'(\psi^{-1}(E_1[\psi(\tilde{c}_2)])) \frac{E_1[\psi'(\tilde{c}_2)(1 + \tilde{r})]}{\psi'(\psi^{-1}(E_1[\psi(\tilde{c}_2)]))} = u'(c_1)$$

For expository purposes, Kimball and Weil treat this equation as a supply-and-demand system in  $s_1$ , where the left-hand side reflects the agent's future demand for savings, and the right-hand side reflects his or her willingness to supply of savings out of current wealth.

Their intuition is also useful here for illustrating the type of data we would need to identify the model. In particular, we see that any observed savings amount will be an intersection point of these supply and demand curves. To identify these curves, we thus need to have data on the savings response to several exogenous shifts in these curves. We can generate such exogenous shifts by varying the scenarios presented to subjects during the experiment. We have three channels of variation at our disposal:

- the type of uncertainty subjects encounter (*i.e.*, in future income or return),
- the probabilities associated with these random events, and
- the payoff level.

In addition, we can construct these shifts in a manner beneficial to identifying prudence using the results of Eeckhoudt and Schlesinger (2008). There, the authors provide some necessary and sufficient conditions for savings to increase in response to changes in risk when vNM utility is used

in this two-period model. We provide a similar set of conditions below for SKP preferences. For the reader who does not wish to peruse the whole of these proofs, we highlight the following main results:

- Corollaries 1 and 2 provide insight into the types of savings patterns that ought to be observed under different types of uncertain events. Decisions over income gambles can help to identify the sign of the derivatives of  $v$ , and decisions over interest-rate gambles can help to identify a range for the coefficients of prudence  $v$ . Variation in mean-preserving spreads of these gambles can locally identify  $v'''$ , and variation in downside risks can identify the local rate of change of  $v'''$ .
- Proposition 3 shows that these variations in scenarios can also identify  $\psi$ . The return gambles provide a particularly clean, linearly-separable substitution effect arising solely from the risk preference. Hence, variation in savings rates in these scenarios can potentially provide a relatively clear separation of risk attitudes and intertemporal attitudes, particularly if the latter are also strongly identified by income gambles.

Even with these helpful theoretical identification results in hand, it is not immediately obvious how to empirically identify  $v$  and  $\psi$ . For example, it is not clear how many scenarios should be presented, or what range of payoffs should be used, or how much of a time lag should occur between periods 1 and 2. These design elements will be explored further via the experimental sessions.

### 3.1 Comparative Statics with Changes in Income and Return Risk

By studying comparative statics of the savings decision with respect to income and return risk, we can derive some statements about the savings patterns that ought to be observed in this two-period setting under each type of scenario. Eeckhoudt and Schlesinger (2008) perform such an analysis under the assumption of vNM utility; we proceed in a similar fashion under the assumption of SKP preferences. Importantly, the results of these comparative statics depend upon the source of the risk, and are generally different from conditions that arise from the simple addition of risk to certainty (as in Kimball and Weil's analysis). Eeckhoudt and Schlesinger make frequent use of

“increases in  $N^{\text{th}}$ -degree risk” between two gambles (Ekern 1980). This definition of an increase in risk requires (a) that moments  $1, \dots, N - 1$  be identical in both gambles, and (b) that one gamble stochastically dominate the other via  $N^{\text{th}}$ -order stochastic dominance (NSD). A mean-preserving spread is an example of second-degree increase in risk, while an increase in downside risk is an example of a third-degree increase.

Risk can enter our model from either the second-period income or the second-period return. Recall that the first-order condition is

$$U'(s) = -u'(y_1 - s) + \beta v'(\cdot) \frac{E_1 \left[ \psi' \left( \tilde{y}_2 + s\tilde{R} \right) \tilde{R} \right]}{\psi' \left( \psi^{-1} \left( E_1 \left[ \psi \left( \tilde{y}_2 + s\tilde{R} \right) \right] \right) \right)} = 0 \quad (12)$$

where  $\tilde{R} \equiv 1 + \tilde{r}$  is the gross return. Given our prior assumptions about strict concavity of  $u$ ,  $v$ ,  $\psi$ , and the absolute risk tolerance of  $\psi$ , the second-order condition

$$U''(s) = u''(y_1 - s) + \beta v''(\cdot) \left\{ \frac{E_1 \left[ \psi' \left( \tilde{y}_2 + s\tilde{R} \right) \tilde{R} \right]}{\psi' \left( \psi^{-1} \left( E_1 \left[ \psi \left( \tilde{y}_2 + s\tilde{R} \right) \right] \right) \right)} \right\}^2 < 0 \quad (13)$$

ensures that condition (12) has a unique solution for  $s$ . The solution is positive for positive expected (net) returns, which we assume to hold. The following equivalence statement will be important in following proofs.<sup>13</sup>

**Definition 1 *NSD equivalence*** (cf. *Eeckhoudt and Schlesinger*): Given two random variables  $\tilde{z}_i$ ,  $i = a, b$ , the following two statements are equivalent.

1. (i)  $\tilde{z}_a$  dominates  $\tilde{z}_b$  via NSD.
2. (ii)  $Ef(\tilde{z}_a) \leq Ef(\tilde{z}_b)$  for any arbitrary function  $f$  such that  $\text{sgn}(f^{(n)}(t)) = (-1)^n$  for all  $n = 1, 2, \dots, N$ .

Let  $s_{y_{2,i}}^*$  denote the optimal solution to equation (12) when the return  $r$  is certain and second-period income is  $\tilde{y}_2 = \tilde{y}_{2,i}$  for  $i = a, b$ . Associate in the NSD equivalence  $\tilde{z}_i$  with second-period

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<sup>13</sup>Where unambiguous, we will also use the notation  $f^{(n)}(t) \equiv \frac{\partial^n f(t)}{\partial t^n}$  in this section.

income  $\tilde{y}_{2,i}$  and function  $f$  with  $v'(\psi^{-1}(E_1\psi(t)))$ . Then, according to the NSD equivalence,

$$s_{y_{2,b}}^* \geq s_{y_{2,a}}^* \Leftrightarrow \frac{d}{ds}v(\psi^{-1}(E_1\psi(\tilde{y}_{2,b} + sR))) \geq \frac{d}{ds}v(\psi^{-1}(E_1\psi(\tilde{y}_{2,a} + sR))) \quad (14)$$

if  $\text{sgn}\left(\frac{d^{n+1}v(\cdot)}{ds^{n+1}}\right) = (-1)^n$  for all  $n = 1, 2, \dots, N$ . The following proposition summarizes this result.

**Proposition 1** *Let  $s_{y_{2,i}}^*$  denote the optimal solution to equation (12) for a sure return  $r$  and risky second-period income  $\tilde{y}_2 = \tilde{y}_{2,i}$  for  $i = a, b$ . The following two statements are equivalent.*

1.  $s_{y_{2,b}}^* \geq s_{y_{2,a}}^*$ , if  $\text{sgn}[v^{(n)}(\psi^{-1}(E_1\psi(t)))] = (-1)^{n+1}$  for all  $n = 1, 2, \dots, N + 1$ .
2.  $\tilde{y}_{2,a}$  dominates  $\tilde{y}_{2,b}$  via NSD.

Note that Proposition 1 only provides necessary and sufficient conditions for higher optimal saving to occur in the face of a given stochastic-dominance relationship between alternative second-period incomes. It does not predicate the signs of the derivatives of  $v$ . Using Ekern's definition of an increase in  $N^{\text{th}}$ -degree risk, the conditions of Proposition 1 simplify as follows.

**Corollary 1** *Let  $s_{y_{2,i}}^*$  denote the optimal solution to equation (12) for a sure return  $r$  and risky second-period income  $\tilde{y}_2 = \tilde{y}_{2,i}$  for  $i = a, b$ . The following two statements are equivalent.*

1.  $s_{y_{2,b}}^* \geq s_{y_{2,a}}^*$  if  $\text{sgn}[v^{(N+1)}(\psi^{-1}(E_1\psi(t)))] = (-1)^N$ .
2.  $\tilde{y}_{2,b}$  is an  $N^{\text{th}}$ -degree increase in risk over  $\tilde{y}_{2,a}$ .

Statement 1 in Proposition 1 and Corollary 1 are analogous to Eeckhoudt and Schlesinger's respective conditions on marginal expected utility, but here involve the first derivative of  $\psi$  in the risk-preference adjusted return  $\frac{E_1[\psi'(\cdot)\tilde{R}]}{\psi'(\psi^{-1}(E_1[\psi(\cdot)]))}$  and at least one higher derivative of  $v$ . While the corollary only contains the sign condition for  $n = N + 1$ , the proposition holds for any  $N$  sign conditions on  $v^{(n)}(\cdot)$  for all  $n = 1, 2, \dots, N + 1$ . Given that  $\frac{E_1[\psi'(\cdot)\tilde{R}]}{\psi'(\psi^{-1}(E_1[\psi(\cdot)]))} > 0$ , the conditions prescribe only sign properties of the derivatives of  $v$ , and do not include higher derivatives of  $\psi$ , in contrast to the KW coefficients of prudence.

The necessary and sufficient conditions for changes in return risk to increase optimal saving are more complicated, due to the endogeneity of the riskiness of second-period consumption and the savings choice.<sup>14</sup>

**Proposition 2** *Let  $s_{R_i}^*$  denote the optimal solution to equation (12) for a sure second-period income  $y_2$  and risky gross return  $\tilde{R}_i$  for  $i = a, b$ . The following two statements are equivalent.*

1.  $s_{R_b}^* \geq s_{R_a}^*$  if  $-s \cdot \frac{v^{(n+1)}(\cdot)}{v^{(n)}(\cdot)} \cdot \frac{E_1[\psi'(\cdot)\tilde{R}]}{\psi'(\psi^{-1}(E_1[\psi(\cdot)]))} \geq n$  for all  $n = 1, 2, \dots, N$ .
2.  $\tilde{R}_a$  dominates  $\tilde{R}_b$  via NSD.

**Proof.** Note that condition (14) holds analogously for any sure second-period endowment income  $y_2$  and risky gross return  $\tilde{R} = \tilde{R}_i$ ,  $i = a, b$ , if function  $h(\tilde{R}) \equiv v'(\cdot) \frac{E_1[\psi'(\cdot)\tilde{R}]}{\psi'(\psi^{-1}(E_1[\psi(\cdot)]))}$  satisfies  $\text{sgn}(h^{(n)}(\tilde{R})) = (-1)^n$  for all  $n = 1, 2, \dots, N$ . For induction, consider first  $N = 1$ . With  $\frac{\partial v'(\cdot)}{\partial \tilde{R}} = sv''(\cdot)$  and  $\frac{\partial}{\partial \tilde{R}} \frac{E_1[\psi'(\cdot)\tilde{R}]}{\psi'(\psi^{-1}(E_1[\psi(\cdot)]))} = 1$ ,  $h'(\tilde{R}) = sv''(\cdot) \frac{E_1[\psi'(\cdot)\tilde{R}]}{\psi'(\psi^{-1}(E_1[\psi(\cdot)]))} + v'(\cdot)$ , so that  $h'(\tilde{R}) \leq 0$  is equivalent to  $-s \frac{v''(\cdot)}{v'(\cdot)} \frac{E_1[\psi'(\cdot)\tilde{R}]}{\psi'(\psi^{-1}(E_1[\psi(\cdot)]))} \geq 1$  for  $v'(\cdot) > 0$ . From standard induction arguments, the following formula derives for any  $n > 1$ :  $h^{(n)}(\tilde{R}) = s^n v^{(n+1)}(\cdot) \frac{E_1[\psi'(\cdot)\tilde{R}]}{\psi'(\psi^{-1}(E_1[\psi(\cdot)]))} + ns^{n-1}v^{(n)}(\cdot)$ . Proposition 2 then follows as in Eeckhoudt and Schlesinger (2008: 1334). ■

Again, Ekern's definition of risk simplifies the conditions of Proposition 2.

**Corollary 2** *Let  $s_{R_i}^*$  denote the optimal solution to equation (12) for a sure second-period income  $y_2$  and risky gross return  $\tilde{R}_i$  for  $i = a, b$ . The following two statements are equivalent.*

1.  $s_{R_b}^* \geq s_{R_a}^*$  if  $-s \cdot \frac{v^{(N+1)}(\cdot)}{v^{(N)}(\cdot)} \cdot \frac{E_1[\psi'(\cdot)\tilde{R}]}{\psi'(\psi^{-1}(E_1[\psi(\cdot)]))} \geq N$ .
2.  $\tilde{R}_b$  is an  $N^{\text{th}}$ -degree increase in risk over  $\tilde{R}_a$ .

Statement 1 in Proposition 2 and Corollary 2 are close analogs to the respective conditions in Eeckhoudt and Schlesinger. Thus, under both expected utility and SKP, the conditions involve expressions that resemble coefficients of relative  $N^{\text{th}}$ -degree risk aversion or relative  $N^{\text{th}}$ -degree resistance to intertemporal substitution, only for the case of  $y_2 = 0$  (as assumed by Eeckhoudt and

<sup>14</sup>Selden (1979) studies savings behavior under return risk using his type of recursive preferences. Langlais (1995) treats the case of  $N = 2$ , as does Weil (1990) for generalized isoelastic preferences.

Schlesinger).<sup>15</sup> As in the case of risky second-period income and sure return, the conditions only involve the first derivative of  $\psi$  and particularly depend on the sign properties of the derivatives of  $v$ .

To see how the two settings generate different savings behavior, consider an increase in second-order risk ( $N = 2$ ) in each variable. When the increase arises from second-period income, precautionary saving will increase if the agent is prudent with respect to  $v$ . However, when the increase arises from the second-period return, two effects arise as Eeckhoudt and Schlesinger note, which can be observed via the equation

$$h''(\tilde{R}) = s^2 v'''(\cdot) \frac{E_1 [\psi'(\cdot) \tilde{R}]}{\psi'(\psi^{-1}(E_1[\psi(\cdot)]))} + 2s v''(\cdot)$$

If  $v$  were quadratic with  $v'' > 0$  but  $v''' = 0$ , then  $h''(\tilde{R}) < 0$ . An agent with resistance to intertemporal substitution would save less because saving increases the second-order riskiness of second-period consumption. This constitutes a substitution effect of current consumption for future consumption. If, however, the consumer is prudent with respect to  $v$ , then the additional second-order risk on second-period consumption will induce a precautionary effect. For a net increase in savings to occur, this precautionary effect must dominate the substitution effect. This occurs if and only if the product of optimal saving and the coefficient of prudence of  $v$  exceeds two.

Finally, the change in optimal savings for a scenario element  $\theta_i$  is given by the implicit function theorem:

$$\frac{ds^*}{d\theta_i} = - \frac{\frac{\partial F(s;\theta)}{\partial \theta_i}}{\frac{\partial F(s;\theta)}{\partial s}} \Big|_{s=s^*} \quad (15)$$

where  $F(s;\theta)$  is defined as  $U'(s)$  under the scenario  $\theta$ . Gollier (2001) provides sign conditions on  $ds^*/d\theta_i$  under expected utility, which extend to the case of SKP preferences.<sup>16</sup> We also provide a

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<sup>15</sup>Eeckhoudt and Schlesinger refer to a quantity of the form  $-\frac{tu^{(n)}(t)}{u^{(n-1)}(t)}$  as a measure of “relative  $n^{th}$ -degree risk aversion” and use the established terms of risk aversion, prudence, temperance and edginess for  $n = 2, 3, 4, 5$ . The term “resistance to intertemporal substitution” for coefficients involving two adjacent derivatives of the intertemporal function  $v$  is used by Kimball and Weil.

<sup>16</sup>Because  $\frac{\partial F(s;\theta)}{\partial s} < 0$  (see equation (13)), the sign of  $\frac{ds^*}{d\theta_i}$  in equation (15) is uniquely determined by the numerator of equation (15).

condition summarizing the effect of adding a constant  $\bar{c}$  to the consumption level in each period, which will prove useful later.

**Proposition 3** *Let  $s^*$  denote the optimal solution to equation (12). A marginal increase in an endowment parameter changes optimal savings ceteris paribus as follows.*

$$\text{sgn} \left[ \frac{ds^*}{dy_1} \right] = \text{sgn} [-u''(\cdot)] > 0 \quad (16)$$

$$\text{sgn} \left[ \frac{ds^*}{d\tilde{y}_2} \right] = \text{sgn} \left[ v''(\cdot) \frac{E_1[\psi'(\cdot)\tilde{R}]}{\psi'(\psi^{-1}(E_1[\psi(\cdot)]))} \right] < 0 \quad (17)$$

$$\text{sgn} \left[ \frac{ds^*}{d\tilde{r}} \right] = \text{sgn} \left[ sv''(\cdot) \frac{E_1[\psi'(\cdot)\tilde{R}]}{\psi'(\psi^{-1}(E_1[\psi(\cdot)]))} + v'(\cdot) \right] \begin{matrix} \geq 0 \\ \leq 0 \end{matrix} \text{ for } -s \frac{v''(\cdot)}{v'(\cdot)} \frac{E_1[\psi'(\cdot)\tilde{R}]}{\psi'(\psi^{-1}(E_1[\psi(\cdot)]))} \begin{matrix} \leq 1 \\ \geq 1 \end{matrix} \quad (18)$$

$$\text{sgn} \left[ \frac{ds^*}{d\bar{c}} \right] = \text{sgn} \left[ -u''(\cdot) + \beta v''(\cdot) \frac{E_1[\psi'(\cdot)\tilde{R}]}{\psi'(\psi^{-1}(E_1[\psi(\cdot)]))} \right] < 0 \text{ if} \quad (19)$$

$$\begin{cases} ARA'_\psi < 0 \text{ and } -\frac{v''(\cdot)}{[v'(\cdot)]^2} \text{ weakly increasing} & \text{or} \\ ARA'_\psi, AP'_\psi < 0, ARIS_v \geq ARA_\psi, \text{ and } \varepsilon_v + RRIS_v \leq \varepsilon_\psi + RRA_\psi \end{cases}$$

The sign conditions in Proposition 3 derive from straightforward calculation of all derivatives involved. The qualification in condition (18) comes from Gollier by analogy, and the qualifications in condition (19) stem from Kimball and Weil. As conditions (16) and (17) indicate, optimal savings increase with  $y_1$  but decrease with  $\tilde{y}_2$ , expressing the preference for consumption smoothing. Note that this property solely depends on characteristics of the intertemporal utility functions  $u$  and  $v$ . The sign of the impact of a marginal increase in the interest rate  $\tilde{r}$  depends on the relative strengths of two effects. First, the prospect of being wealthier in the future permits the agent to have a higher consumption today, smoothing consumption over time. This wealth effect is captured by the negative first term on the right-hand side of equation (18). Second, an increase in the interest rate gives an incentive to save more today and thus to substitute future consumption for current consumption due to the reduction of the relative price of future consumption. This substitution effect is expressed by the positive second term. The substitution effect dominates (is dominated by) the wealth effect if the coefficient of relative resistance to substitution of  $v$  with respect to  $s$

is smaller (larger) than 1, as can be seen by factoring out  $v'(\cdot)$ . Finally, the impact of an increase in  $\bar{c}$  is ambiguous, as it hinges on the relative strengths of the two effects stated in conditions (16) and (17).

## 4 Experimental Design and Econometric Analysis

The two-period consumption/savings model and the conditions for increased saving under risk changes of Section 3 guide our experimental design including the construction of scenarios. The design comprises two phases, which are separated by a time delay of several days or weeks. In Phase 1, subjects are presented with a list of scenarios of endowment income  $y_1$  and a future lottery over either Phase-2 endowment  $y_2$  or return  $r$ . In each scenario, a subject chooses the savings amount  $s_1$  out of endowment income  $y_1$ . The scenarios are presented to subjects in a manner similar to a “multiple price list.” (An illustrative list for each of the income and interest tasks can be found in Tables 1 and 2 below.) At the end of Phase 1, two scenarios are picked at random to be paid in full value. Subjects are paid  $c_1 = y_1 - s_1$  for these scenarios. In Phase 2, subjects first complete a set of surveys lasting about as long as Phase 1.<sup>17</sup> The surveys include the Cognitive Reflection Test, the Big-Five Inventory personality test, and demographics. Subjects then resolve uncertainty in  $y_2$  or  $r$  in their scenarios for payment, and are paid  $c_2 = y_2 + s_1(1 + r)$  for these scenarios.

Each scenario belongs to either of two classes of decision tasks which correspond to the two types of random events in the model. In the first task class, the return is fixed at  $r = 20\%$ , and risk enters only from uncertainty in second-period income, which is centered at  $y_1 = E_1(\tilde{y}_2) = \$20$ . In the second task class, incomes are fixed at  $y_1 = y_2 = \$20$ , and risk enters only from uncertainty in the return, which is centered at  $E_1(\tilde{r}_2) = 20\%$ . The risky element of each task is a binary gamble consisting of two prospects of the form  $(z; Pr(z))$ , where  $z \in \{y_2, r\}$ . For notational convenience, we denote  $z^h$  as the higher and  $z^l$  as the lower of the two outcomes, and associate  $\pi$  with the probability of the higher outcome  $Pr(z^h)$ .

Given this setup, we construct risk changes in the scenarios that may be useful for identifying

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<sup>17</sup>In a recent intertemporal design, Andreoni and Sprenger (2009b) note the importance of equalizing unobserved shadow costs of visiting the laboratory across visits (*e.g.*, time costs).

Kimball-Weil prudence:

- Base case:  $\mathcal{L}_z = \{(z^l; 1 - \pi), (z^h; \pi)\}$
- Mean-preserving spread:  $\mathcal{L}_z = \{(z^l - \eta; 1 - \pi), (z^h + \gamma; \pi)\}$
- Increased downside risk:  $\mathcal{L}_z = \{(z^l + \nu; 1 - \pi + \delta), (z^h + \mu; \pi - \delta)\}$

The two prospects in a base lottery can be altered to satisfy the appropriate risk definition. For a mean-preserving spread, this involves subtracting  $\eta$  from the lower outcome and adding a constant  $\gamma$  to the higher outcome, so that the new gamble has the same mean as the original. For an increase in downside risk, this involves adding a constant  $\delta$  to the lower outcome's probability and subtracting  $\delta$  from the higher outcome's probability, and then adding  $\mu$  and  $\nu$  to the high and low outcomes, so that the original mean and variance are preserved.<sup>18</sup>

In the income tasks, savings should increase with  $\gamma$  if  $v'''(\psi^{-1}(E_1\psi(\tilde{c}_2))) \geq 0$ , and with  $\delta$  if  $v'''(\psi^{-1}(E_1\psi(\tilde{c}_2))) \leq 0$  (Corollary 1). In the interest-rate tasks, savings should increase with  $\gamma$  if  $-\frac{v''''(\cdot)}{v''(\cdot)} \cdot s \cdot \frac{E_1[\psi'(\cdot)(1+\tilde{r})]}{\psi'(\cdot)} \geq 2$ , and with  $\delta$  if  $-\frac{v''''(\cdot)}{v''(\cdot)} \cdot s \cdot \frac{E_1[\psi'(\cdot)(1+\tilde{r})]}{\psi'(\cdot)} \geq 3$  (Corollary 2). Moreover, adding income risk to a setting of certainty raises savings under KW prudence (*cf.* Section 3). To assess these tendencies, the design involves, apart from a degenerate lottery, the exogenous manipulations  $\eta \in \{0, 4, 8, 12, 16\}$  for  $\pi = 0.5$  and  $\pi = 20/(20 - \eta)$ , yielding a total of 9 savings choices involving mean-preserving spreads. Moreover, we pick the scenarios with  $\eta \in \{8, 16\}$  and  $\pi = 0.5$  to construct lotteries with the exogenous manipulations  $\delta \in \{0.1, 0.2, 0.25, 0.3, 0.4\}$ , for a total of 10 savings choices involving increases in downside risk.

In addition to varying the risks and payoffs, we also vary the payoff scale. The idea that risk attitudes can vary with the wealth level is at least as old as Pratt (1964). Rabin (2000) provides evidence that this variation is actually empirically necessary to avoid troublesome predictions about lottery choices at varying scales, but that such variation still leaves some puzzling predictions in the expected-utility framework. Indeed, Holt and Laury (2002) note exactly such a wealth effect when their lottery-choice design is conducted at substantially different payoff levels. Because prudence can potentially vary with the income level, and because the higher income levels are more likely

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<sup>18</sup>This implies that  $\mu_{1,2} = (z^h - z^l) \left( \pi \pm \sqrt{\frac{\pi(1-\pi)}{1-\pi+\delta}} \right)$ , and  $\nu_{1,2} = (z^h - z^l) \left( \pi \pm \sqrt{\pi(1-\pi)\frac{1-\pi+\delta}{\pi-\delta}} \right)$ .

Scenario	Today's Income	Tomorrow's Income	Tomorrow's Interest Rate	How Much to Save Today?
1	\$20.00	\$16.00 with probability 0.5 \$24.00 with probability 0.5	20%	\$ -----
2	\$20.00	\$8.00 with probability 0.4 \$28.00 with probability 0.6	20%	\$ -----
3	\$20.00	\$9.52 with probability 0.7 \$44.44 with probability 0.3	20%	\$ -----

Table 1: Decision table for example tasks involving income lotteries,  $x = 1$  scaling.

Scenario	Today's Income	Tomorrow's Income	Tomorrow's Interest Rate	How Much to Save Today?
1	\$20.00	\$20.00	16.0% with probability 0.5 24.0% with probability 0.5	\$ -----
2	\$20.00	\$20.00	12.0% with probability 0.6 32.0% with probability 0.4	\$ -----
3	\$20.00	\$20.00	17.3% with probability 0.9 44.0% with probability 0.1	\$ -----

Table 2: Decision table for example tasks involving interest-rate lotteries,  $x = 1$  scaling.

to reflect our behavior of interest (prudential savings), we believe it is important to collect savings decisions at a variety of income levels. Thus, the design also involves collecting similar decisions for payoffs scaled by factors  $x \in \{1, 2.5, 5, 7.5, 10\}$ . These lead to gambles with the same qualitative risk characteristics, but substantially different payoffs.

To concretize the content of the list, we consider a case in which five tasks with income gambles are always followed by five gambles with return gambles. (In the experiment, the “return” is called an “interest rate” for clarity.) Moreover, the payoff scaling changes every five or ten tasks. The first half of the list starts with  $x = 1$  and then moves to  $x = 10$  and  $x = 7.5$ , while the second half starts again with  $x = 1$  and moves to  $x = 2.5$  and  $x = 5$ . Presenting decisions to subjects in a semi-structured fashion may lead to fewer inconsistent choices than if the scenarios are presented in a random order.<sup>19</sup> This particular elicitation procedure generates 60 scenarios and associated savings decisions.

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<sup>19</sup>We thank Charlie Holt for this helpful suggestion.

## 4.1 Econometric Analysis

We use each subject's savings data in order to estimate the parameters of our SKP-enhanced Euler equation (6) at the individual level. For a particular model parametrization  $\omega = (\beta, \alpha_\psi, \rho_\psi, \alpha_v, \rho_v)$ , Euler equation (6) implicitly defines a unique savings amount  $s_{ij}^*(\omega)$  for each scenario  $j$  faced by subject  $i$ . Our econometric strategy attempts to match this prediction with the quantity  $s_{ij}$  actually observed. In particular, we minimize the sum of squared prediction errors  $\eta_{ij} = s_{ij} - s_{ij}^*(\omega)$ ,

$$\hat{\omega}_i = \arg \min_{\omega} \sum_{j=1}^{J_i} \eta_{ij}^2 = \sum_{j=1}^{J_i} (s_{ij} - s_{ij}^*(\omega))^2$$

over all  $J_i$  scenarios faced by this subject. The asymptotic covariance matrix for this well-known nonlinear least-squares estimator is  $\sigma_{i\eta}^2 (S_i' S_i)^{-1}$ , where  $\sigma_{i\eta}^2$  is the variance of  $\eta_{ij}$  and  $S_i$  is the Jacobian of  $s_i = (s_{i1}^*(\omega), \dots, s_{iJ_i}^*(\omega))$ . For each subject, we had difficulty numerically approximating  $\nabla s_{ij}^*(\omega)$  in some scenarios, making  $S_i' S_i$  generally ill-behaved and unsuitable for inversion. Hence, we do not provide standard errors for any of our estimates. In light of the poor performance of the asymptotic estimator, a bootstrap estimator is probably a more robust method for obtaining the covariance of  $\hat{\omega}_i$ .

We encountered a few identification issues when estimating this model. First, the utility discount factor  $\beta$  is not very well identified, a result that is probably not surprising given the short time intervals used thus far (*cf.* Section 5). We hence constrain  $\beta = 1$  throughout, leaving us with a four-parameter specification. Second, the model exhibited very poor fit when we defined ‘‘consumption’’ entirely within the context of the experiment. The model performed much better when we added an additional consumption quantity  $\bar{c}$  in each period, so that

$$\begin{aligned} c_1 &= \bar{c} + y_1 - s_1 \\ \tilde{c}_2 &= \bar{c} + \tilde{y}_2 + (1 + \tilde{r}) s_1 \end{aligned}$$

This result could be interpreted in a few ways. First, it may reflect a specification error in our model's structure. If true, the source of the error must closely reflect the risk and intertemporal

changes generated by the model via uniformly shifting consumption by  $\bar{c}$ , which would be a very peculiar kind of error. Alternatively, it could imply that subjects consider the savings problem from a context larger than the laboratory, so that  $\bar{c}$  reflects a background consumption level they either have or anticipate having during each period. Indeed, several of the scenarios involve monetary values that are quite high for our student cohort, and they may have seriously considered the consumption-smoothing possibilities implied by these scenarios. If this is the correct interpretation, then the consumption smoothing we observe is not solely generated within the laboratory; the laboratory has spilled partially into the field. Indeed, we might be more assured that we are estimating risk and intertemporal characteristics that are relevant to field decisions.

Importantly, our experiment still generated an unanticipated, controlled shock to subjects' consumption possibilities set, so the responses we observe probably only reflect this exogenous source of variation and not any field elements beyond the background consumption. In addition, because our subject pool consists of in-session undergraduates, it is unlikely that they would have had an opportunity to reassess their second-period background consumption as a result of the experiment. And, the theoretical results from the previous section are impervious to the addition of a constant  $\bar{c}$ , so our scenario selection still has value for identifying prudence. However, the comparative static exercise on initial wealth in equation (19) shows that the interpretation of the outcomes will likely vary with  $\bar{c}$ .

Thus, if our conjecture about the importance of background consumption is valid, we have an additional question of how to empirically address  $\bar{c}$ . This is not without some concern, because it is possible to generate all manner of risk and intertemporal attitudes by arbitrarily scaling  $\bar{c}$  for given data. The best option is probably to conduct a detailed survey about subjects' current and anticipated consumption during the first phase of the experiment, but we unfortunately did not anticipate this requirement for these sessions. A second option is to treat  $\bar{c}$  as an estimatable parameter, but it is not clear how the savings data would identify it. A further option is to impute a value for  $\bar{c}$ . Because our subjects were in-session undergraduate students, it is likely that they had a relatively limited and consistent set of consumption expenditures. In addition, they were probably most concerned about consumption over a relatively short interval. The University of Virginia

Session	Scenarios	Interval	Subjects	Male/Female	Avg. Earnings
1	30	2 days	5	1/4	\$326.26
2	54	7 days	5	3/2	\$331.08
3	60	14 days	5	1/4	\$193.49
4	60	14 days	5	4/1	\$280.95

Table 3: Session description.

publishes estimates of the expenditures that an undergraduate can anticipate during an academic year,<sup>20</sup> and we imputed a monthly consumption value by summing the total non-tuition elements of this estimate and dividing by the number of months in a standard two-semester academic year.<sup>21</sup> Under this method, our estimate of monthly background consumption is \$1620. Finally, because of the variation in the time intervals in different sessions, we tailored  $\bar{c}$  according to the actual interval. The final imputed values of  $\bar{c}$  are  $\$1620/15 = \$108$  in Session 1,  $\$1620/4 = \$405$  in Session 2, and  $\$1620/2 = \$810$  in Sessions 3 and 4.

We estimated the model using the simulated annealing algorithm, a stochastic search process that is relatively robust to the presence of local minima and nonconvexities. We began the search at linear specifications for  $v$  and  $\psi$ , and stopped it after 10,000 random points were evaluated in the parameter space. For most subjects, the algorithm converged upon a minimum within a few thousand iterations, implying that our initial assumption about the existence of KW prudence is probably safe. In three cases, it did not find an improved point. In one of these cases, the subject always saved the all of the first-stage income; the other two exhibited small variations in savings decisions that were probably not sufficient to identify the savings function.

## 5 Data and Results

Our empirical results are based on four sessions with five undergraduate subjects each that we conducted at the Veconlab Experimental Economics Laboratory at the University of Virginia in April and May 2010. Table 3 describes the sessions. We scheduled, when possible, the sessions so that subjects arrived on the same day of the week on each visit, in an attempt to keep the external

<sup>20</sup>These estimates are available at <http://www.virginia.edu/financialaid/estimated.php>.

<sup>21</sup>This total includes room, board, and various discretionary items, but not tuition and fees.

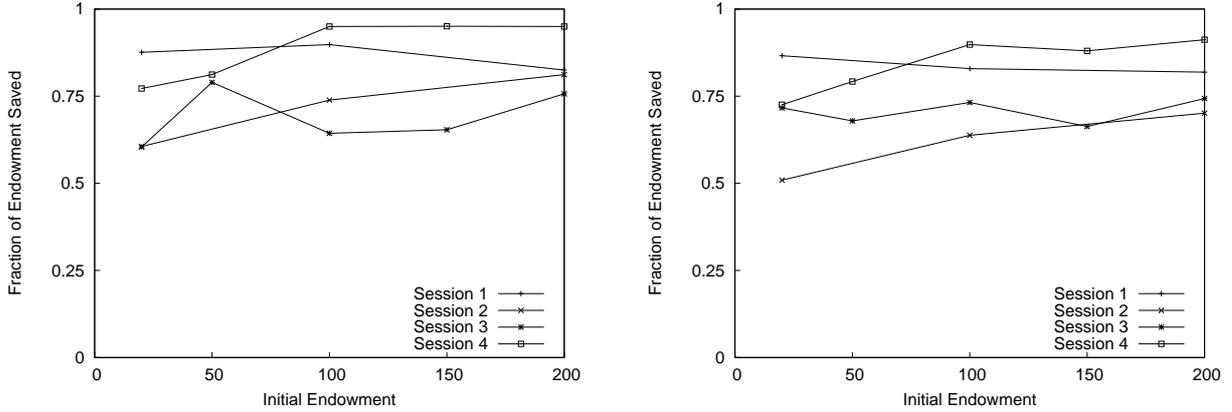


Figure 1: Savings rates by  $y_1$ , for income tasks (left) and interest tasks (right).

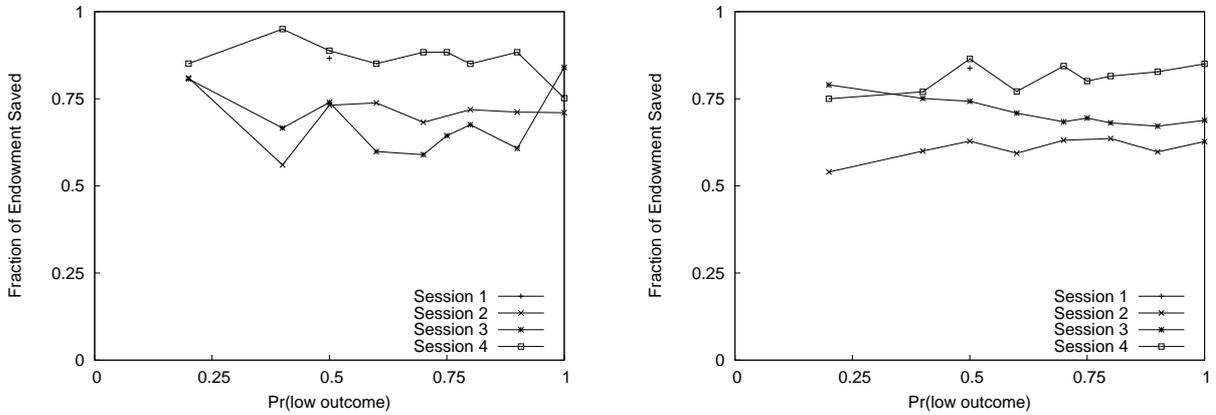


Figure 2: Savings rates by  $1 - \pi$ , for income tasks (left) and interest tasks (right).

influence of regular wage payments constant. Because the dollar amounts paid to subjects can be high, we ran the sessions in very quick succession (within two days) so that there would be limited time for word-of-mouth discussion about the experiment. In the first session, we used 30 scenarios and a lag of 2 days. These scenarios did not include any downside-risk manipulations. The second session involved 54 scenarios with both mean-preserving spreads and downside risk increases, and a lag of 7 days. The last two sessions used the full 60 scenarios described above and lags of 14 days each. Average payments routinely exceeded \$200, because subjects in this small sample received favorable draws of the random elements.

Figure 1 plots the average savings rates in these sessions by initial endowment, and Figure 2 plots them by probability of the low outcome in the lottery. On average, subjects appear to save a good deal: average savings rates are always above 50%. Interestingly, subjects appear to be

Subject	Session	$\alpha_\psi$	$\rho_\psi$	$\alpha_v$	$\rho_v$
1	1	–	–	–	–
2	1	0.24	0.64	0	0.50
3	1	0.12	2.90	0	0.31
4	1	$\epsilon$	0.01	0	0.42
5	1	0.01	0	0	0.41
6	2	0.08	1.03	$\epsilon$	0.45
7	2	0.11	1.51	$\epsilon$	0.45
8	2	$\epsilon$	0	0.25	1.39
9	2	$\epsilon$	0	0.20	1.26
10	2	0.02	0.07	0	0.73
11	3	0.02	0.76	0.21	1.25
12	3	0.06	0.40	0.01	2.13
13	3	0	0	0.03	0.70
14	3	0.13	0.03	0.03	1.99
15	3	0.07	0.32	0.01	0.67
16	4	0.23	0.58	0.03	0.89
17	4	0.23	1.47	0.01	0.49
18	4	$\epsilon$	0.59	0.18	1.17
19	4	–	–	–	–
20	4	–	–	–	–

Table 4: Estimates of the model parameters by subject. An estimate of  $\epsilon$  denotes an estimate that is numerically close to 0. A blank estimate (–) denotes a numerical optimization that did not converge after 10,000 simulated points.

behaving about the same in the return scenarios as in the income scenarios. Conditional on a wealth level, an income scenario is in some sense riskier than a return scenario, because subjects can always choose to save nothing in a return scenario and be assured of two relatively high payments. Hence, they may save more in an income scenario than in the interest scenario. The same consistency is apparent when examining savings rates from the perspective of the probability of the low outcome.

## 5.1 Individual SKP Estimates

The data averages in Figures 1 and 2 give some indication of the average propensity to save, but they do not represent the heterogeneity in decisions. Table 4 provides the estimates of the best-fitting parametrization for each subject that derive with our econometric methodology described in Subsection 4.1. A few features are immediately obvious. First, resistances to intertemporal

Sessions 1-4	Increasing <i>RRA</i>	Decreasing <i>RRA</i>	Constant <i>RRA</i>	Count
Increasing <i>RRIS</i>	2	1		3
Decreasing <i>RRIS</i>	3		4	7
Constant <i>RRIS</i>	2	3	2	7
Count	7	4	6	17
Sessions 3 & 4	Increasing <i>RRA</i>	Decreasing <i>RRA</i>	Constant <i>RRA</i>	Count
Increasing <i>RRIS</i>	2	1		3
Decreasing <i>RRIS</i>	3		2	5
Constant <i>RRIS</i>				
Count	5	1	2	8

Table 5: Breakdown of relative risk aversion and relative resistance to intertemporal substitution.

substitution in Sessions 1 and 2 appear to be usually constant and smaller in magnitude than in Sessions 3 and 4. This could imply that the time delay was not sufficiently long in the first two sessions to create a meaningful intertemporal tradeoff. (In other words, the two stages may not have corresponded to separate evaluation periods.) In addition, subjects in Sessions 1 and 2 also tend to have lower levels of risk aversion, with quite a few instances of risk neutrality. These initial sessions did not involve as many scenarios and may not adequately generate enough variation for estimating risk attitudes either.

The breakdown for *RRA* and *RRIS* in our experimental cohort are provided in Table 5. Cases that did not exhibit model convergence are omitted from the breakdown, and  $\epsilon$  values are treated as 0. In the full cohort, 18% of subjects exhibited IRRIS, 41% exhibited DRRIS, and 41% exhibited CRRIS. In addition, 41% exhibited IRRA, 24% exhibited DRRA, and 35% exhibited CRRA. This picture, however, changes substantially for Sessions 3 and 4. Across this sub-sample, 38% exhibit IRRIS, 62% exhibit DRRIS, and 0% exhibit CRRIS. And, 62% exhibit IRRA, 13% exhibit DRRA, and 25% exhibit CRRA. The shifts of frequency (a) away from CRRIS and (b) towards IRRA strongly suggest that Sessions 3 and 4 are generating different behavior. Of course, this result occurs in a small sample, so it is difficult to provide statistical support for this effect.

Finally, these estimates allow us to estimate the degree of prudence in our subjects. Recall that the functional form for relative KW prudence is

$$RP(c) = RRA_{\psi}(c) \left( 1 + \frac{\varepsilon_{\psi}(c)}{RRIS_v(c)} \right) = RRA_{\psi}(c) \left( 1 + \frac{RP_{\psi}(c) - RRA_{\psi}(c)}{RRIS_v(c)} \right)$$

Subject	Session	$RP$	$RRA_\psi$	$\varepsilon_\psi$	$RP_\psi$	$RRIS_v$	$\varepsilon_v$	$RP_v$	$RP_\psi - RP_v$
1	1	–	–	–	–	–	–	–	–
2	1	4.37	1.93	0.76	2.69	0.60	0.91	1.52	1.18
3	1	3.50	2.90	$1+\epsilon$	3.90	4.86	0.35	5.22	-1.32
4	1	0.13	0.11	0.10	0.21	0.72	0.76	1.48	-1.27
5	1	1.08	1.08	0	1.08	0.73	0.74	1.47	-0.39
6	2	3.28	1.10	$1+\epsilon$	2.10	0.50	0.94	1.46	0.65
7	2	4.73	1.52	$1+\epsilon$	2.52	0.47	0.97	1.45	1.07
8	2	3.24	3.24	0	3.24	1.41	1.01	2.42	0.82
9	2	0.81	0.81	0	0.81	1.30	1.01	2.31	-1.50
10	2	6.00	5.39	0.08	5.47	0.73	1	1.73	3.74
11	3	1.51	0.86	0.97	1.83	1.29	1.01	2.30	-0.47
12	3	4.55	3.74	0.46	4.20	2.13	$1+\epsilon$	3.13	1.07
13	3	0	0	–	–	0.92	0.93	1.85	–
14	3	87.48	86.16	0.03	86.19	1.99	$1+\epsilon$	2.99	83.20
15	3	10.19	6.97	0.35	7.32	0.76	0.96	1.72	5.60
16	4	7.35	4.41	0.64	5.05	0.95	0.99	1.95	3.10
17	4	3.35	1.48	$1+\epsilon$	2.48	0.79	0.80	1.60	0.88
18	4	1.08	0.59	$1-\epsilon$	1.59	1.23	1.08	2.24	-0.64
19	4	–	–	–	–	–	–	–	–
20	4	–	–	–	–	–	–	–	–

Table 6: Point estimates of the components of  $RP$  in the neighborhood of  $\bar{c}$  by subject. A coefficient of  $1 \pm \epsilon$  denotes an estimate that is numerically close to 1.

implying that we need to compute  $RP_\psi$ ,  $RRA_\psi$ , and  $RRIS_v$ .<sup>22</sup> Table 6 presents point estimates of relative KW prudence and its risk and intertemporal components for each subject, evaluated at the applicable  $\bar{c}$ .  $RRA$  is greater than 1 in 12 subjects (71%), and greater than 5 in 5 subjects (29%). Subjects 13 and 14 are outliers in this cohort, one exhibiting risk-neutrality and the other extreme risk aversion.  $RRIS$  is less than 1 in 10 subjects (59%), and greater than 1 in the remaining 7 subjects (41%). The extremes of these estimates are 0.47 and 4.86. The elasticity  $\varepsilon_\psi$  is equal to 1 for 5 subjects (29%), and less than 1 for the remaining 12 subjects (71%). The elasticity  $\varepsilon_v$  is equal to 1 for 3 subjects (17%), less than 1 for 10 subjects (59%), and greater than 1 for 4 subjects (24%). The fact that these elasticities are not usually equal to 1 implies that the increasing or decreasing shape of a subject's  $RRA$  and  $RRIS$  is indeed an important component of his or her risk and intertemporal preferences. (Recall that relative prudence is simply constant at  $1 + \rho$  for power utility, the case in which  $\varepsilon = 1$ .)

Table 6 also shows that risk-domain prudence is stronger than intertemporal-domain prudence for 10 subjects (59%). The correlation coefficient for the two attitudes is 0.24, indicating a moderately positive relationship between risk and intertemporal prudence in an average subject. The estimates of KW prudence range from 0 to 87.48, and are typically less than 5 (76%).

After eliminating the two outliers, we find that the average  $RRA$  is 2.06, the average  $RRIS$  is 1.34 (implying that the elasticity of intertemporal substitution is  $1/1.34 \approx 0.75$ ), and the average  $RP$  is 3.90. These averages, however, mask a good deal of heterogeneity: the standard deviations of these estimates are 1.58 for  $RRA$ , 1.27 for  $RRIS$ , and 2.82 for  $RP$ . Hence, using an average risk attitude to generate savings amounts (*e.g.*, via a representative-agent analysis) would yield savings rates that many subjects would probably not prefer.

## 6 Discussion

Our threefold procedure for analyzing prudence in an intertemporal context – an interrelated web of theory, experiment, and structural estimation – appears to have the ability to elicit and identify

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<sup>22</sup>If subjects are risk neutral,  $RP_\psi$  and  $RRA_\psi$  are identically zero, making  $RP$  zero in that case. And, if subjects have no resistance to intertemporal substitution,  $RP$  explodes. A few of our subjects, particularly in Sessions 1 and 2, exhibit at least one of these characteristics.

Subject	Session	$\alpha_\psi$	$\rho_\psi$	$\alpha_v$	$\rho_v$
1	1	–	–	–	–
2	1	0.05	0.14	0.01	0.27
3	1	0.04	0.22	0.04	1.49
4	1	0.11	1.05	0.05	0.58
5	1	0	0	0.23	1.95
6	2	0.04	0.46	0	1.50
7	2	0.06	0.40	0.01	0.91
8	2	0.01	$\epsilon$	0.01	0.52
9	2	0.03	0.33	0.11	0.53
10	2	0.04	0.19	0.01	0.42
11	3	0.04	1.87	0.09	2.27
12	3	0.03	0.24	0.14	0.58
13	3	0	0.28	0.02	1.59
14	3	0.07	0.12	0.05	0.85
15	3	0.05	0.25	0.06	1.25
16	4	0.11	0.42	0.10	1.63
17	4	0.03	2.46	0.03	1.00
18	4	0.10	0.99	0.09	0.64
19	4	–	–	–	–
20	4	–	–	–	–

Table 7: Estimates of the model parameters by subject for  $\bar{c} = \$1620$ . A value of  $\epsilon$  denotes a number that is numerically close to 0.

prudence in laboratory subjects. We find estimates that are generally plausible, but exhibit a good deal of between-subject variation.

Clearly, the weakest link in our strategy is the fact that we do not have a good grasp on how closely the laboratory and the field are interacting within our subjects. Laboratory actions certainly appear to incorporate some non-laboratory context, so it is essential to understand the external factors as well. To illustrate the sensitivity of our method to this aspect of the model, we provide alternative estimates of risk and intertemporal preferences assuming a common background consumption level  $\bar{c} = \$1620$  in Table 7. Comparing these estimates to those in Table 4 (tailored to a session-specific  $\bar{c}$ ), we find that many have changed substantially. Probably the best way to address this issue is to survey subjects about their current and estimated consumption expenditures at the end of the first stage, and then verify these again as part of the second stage.

We also find that a time delay of a few days and perhaps even a few weeks may not activate the desired risk and intertemporal attitudes we wish to study. This is evident in the complete

lack of empirical identification in the savings data of three of our subjects. Subjects may mentally account the payments as pure windfalls when such payments are large and closely spaced, which may not lead to the desired risk and intertemporal tradeoffs. Probably the best alteration to our design in such a case is to increase both the risk and the time interval.

Interestingly, our study finds a way to reconcile the IRRA results of Holt and Laury (2002) with the CRRIS or DRRIS that is frequently observed in intertemporal problems such as consumption smoothing. Indeed, in the best parametrization of our experiment, the risk estimates appear to be moving primarily towards IRRA, and the intertemporal estimates appear to be moving primarily towards DRRIS. The reason, to use the words of a recent study, is that “risk preferences are not time preferences” (Andreoni and Sprenger 2009b), and so it is possible for subjects to hold both attitudes simultaneously.

This design could potentially be used to classify individuals according to their risk and intertemporal characteristics. The empirical density of such estimates could be used in models for which the distribution of such attitudes is important, such as those mentioned in the literature section. Future research to refine the design and verify its consistency could provide insight into population-level risk and intertemporal preferences.

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# A Experiment Instructions

## Part 1

### Overview

This experiment consists of two parts:

- Part 1 will take place **today**.
- Part 2 will take place **between 14 and 15 days from now**.

In Part 1 of this experiment, you will make a series of decisions that involve monetary amounts. You will receive a real cash payment that is based upon a subset of these decisions.

The decisions you make today will affect the cash payments you receive in Parts 1 and 2 of this experiment. However, you will only receive the cash payment corresponding to Part 1 today. To receive the Part 2 cash payment, you must return for Part 2 of the experiment at the appropriate time.

In Part 2 of this experiment, you will not make any decisions. Instead, you will complete a series of questionnaires, and receive your Part 2 cash payment.

### Scenarios

Scenario	<u>Part 1</u> Endowment	<u>Part 2</u> Endowment	Interest Rate	<b>Part 1</b> <b>Endowment</b> <b>to Save:</b>	<b>Part 2</b> <b>Outcome</b>
1	\$1.00	\$0.25 with probability 0.25 \$1.75 with probability 0.75	10%	\$ ____	TBD
2	\$1.00	\$1.00	5.00% with probability 0.5 20.00% with probability 0.5	\$ ____	TBD

The tables above are examples of the types of scenarios that you will encounter in this experiment. These examples are for illustrative purposes only; the scenarios you actually encounter may be different.

Each scenario consists of **3 given elements**:

- The **Part 1 Endowment** is an amount of money that is paid to you in Part 1. This amount does not involve any random events.
- The **Part 2 Endowment** is an amount of money that is paid to you in Part 2. This amount may have a random component. For example, the Part 2 Endowment is random in Scenario 1 above, but not in Scenario 2.
- The **Part 2 Interest Rate** is an interest rate that is applicable in Part 2. This rate may also have a random component. For example, the Part 2 Interest Rate is random in Scenario 2 above, but not in Scenario 1.

Each scenario also contains **1 decision** to be made:

- The **Part 1 Savings Amount** is an amount of money that you can save from your Part 1 Endowment. Amounts saved in Part 1 are paid with interest in Part 2.

In each scenario, the payments for Parts 1 and 2 are as follows:

- **Part 1 Payment** = Part 1 Endowment – Part 1 Savings
- **Part 2 Payment** = Part 2 Endowment + Part 1 Savings + (Part 1 Savings × Part 2 Interest Rate)

Example 1: Suppose in Scenario 1 above you chose to save half of the Part 1 Endowment. Suppose also that the better of the two Part 2 outcomes occurs. Then, your payments would be:

- **Part 1 Payment** =  $\$1.00 - \$0.50 = \$0.50$
- **Part 2 Payment** =  $\$1.75 + \$0.50 + (\$0.50 \times 0.1) = \$2.30$

Example 2: Suppose in Scenario 2 above you chose to save half of the Part 1 Endowment. Suppose also that the worse of the two Part 2 outcomes occurs. Then, your payments would be:

- **Part 1 Payment** =  $\$1.00 - \$0.50 = \$0.50$
- **Part 2 Payment** =  $\$1.00 + \$0.50 + (\$0.50 \times 0.05) = \$1.53$

## Payoffs

After you make a decision in all 60 scenarios, you will be prompted to randomly draw 2 of them. These scenarios you draw are the sole basis of your cash payment. The rest of the scenarios will have no bearing on your cash payment.

You will draw scenarios using the “**Random Scenario**” program below. This program continuously generates random numbers between **1** and **60**, with equal probability. Once you click the “Draw Scenario” button, the program will stop, and the number that was last generated will be shown. This number is a random scenario draw.

<b>Random Scenario 1:</b> Scenario <input type="text"/> <input type="button" value="Draw Scenario"/>
<b>Random Scenario 2:</b> Scenario <input type="text"/> <input type="button" value="Draw Scenario"/>

When it is time to draw scenarios, you can click the “Draw Scenario” button whenever you feel ready. (You can try this now if you like.)

Important Note: The program generates new numbers very quickly (hundreds of times per second), so it is not possible for you to time your selection to choose any particular scenario.

When you finish drawing your scenarios, your Part 1 Payment for these scenarios will be computed, using the decisions you made. Your total cash payment today will be equal to the sum of these Part 1 Payments.

The corresponding Part 2 Payments will be computed when you return for Part 2. If the scenarios you drew involve a random Part 2 component, those outcomes will be determined when you return, using a similar random-selection process.

## Part 2

### Overview

This experiment consists of two parts:

- Part 1 took place **between 14 and 15 days ago**.
- Part 2 will take place **today**.

In Part 1, you made a series of decisions that involved two payments. You received the first payment at the end of Part 1.

In Part 2 today, you will first complete a series of questionnaires. After you have completed these questionnaires, you will finalize your decisions from Part 1, and receive the second payment.

### Scenarios

Scenario	Part 1 Endowment	Part 2 Endowment	Interest Rate	Part 1 Endowment to Save:	Part 2 Outcome
1	\$1.00	\$0.25 with probability 0.25 \$1.75 with probability 0.75	10%	\$ _____	TBD
2	\$1.00	\$1.00	5.00% with probability 0.5 20.00% with probability 0.5	\$ _____	TBD

As a reminder, in Part 1 you faced a number of scenarios similar to the ones above.

Each scenario consists of **3 given elements**:

- The **Part 1 Endowment** is an amount of money that is paid to you in Part 1. This amount does not involve any random events.
- The **Part 2 Endowment** is an amount of money that is paid to you in Part 2. This amount may have a random component. For example, the Part 2 Endowment is random in Scenario 1 above, but not in Scenario 2.
- The **Part 2 Interest Rate** is an interest rate that is applicable in Part 2. This rate may also have a random component. For example, the Part 2 Interest Rate is random in Scenario 2 above, but not in Scenario 1.

Each scenario also contains **1 decision** to be made:

- The **Part 1 Savings Amount** is an amount of money that you can save from your Part 1 Endowment. Amounts saved in Part 1 are paid with interest in Part 2.

In each scenario, the payments for Parts 1 and 2 are as follows:

- **Part 1 Payment** = Part 1 Endowment – Part 1 Savings
- **Part 2 Payment** = Part 2 Endowment + Part 1 Savings + (Part 1 Savings × Part 2 Interest Rate)

Example 1: Suppose in Scenario 1 above you chose to save half of the Part 1 Endowment. Suppose also that the better of the two Part 2 outcomes occurs. Then, your payments would be:

- **Part 1 Payment** =  $\$1.00 - \$0.50 = \$0.50$
- **Part 2 Payment** =  $\$1.75 + \$0.50 + (\$0.50 \times 0.1) = \$2.30$

Example 2: Suppose in Scenario 2 above you chose to save half of the Part 1 Endowment. Suppose also that the worse of the two Part 2 outcomes occurs. Then, your payments would be:

- **Part 1 Payment** =  $\$1.00 - \$0.50 = \$0.50$
- **Part 2 Payment** =  $\$1.00 + \$0.50 + (\$0.50 \times 0.05) = \$1.53$

## Payoffs

After you made a decision in all 60 scenarios, you were prompted to randomly draw 2 of them. These scenarios you drew are the sole basis of your cash payment. The rest of the scenarios have no bearing on your cash payment.

Because some of the scenarios involve a random component in Part 2, it is necessary to determine which of the possible outcomes will actually be used in your payment. You will draw these outcomes using the “Random Outcome” program below. This program continuously generates outcomes with the applicable probability. Once you click the “Draw Outcome” button, the program will stop, and the number that was last generated will be shown. This number is a random outcome draw.

Scenario 1, Part 2 Endowment

\$0.25 with probability 0.25

\$1.75 with probability 0.75

**Random Outcome:** \$

Scenario 2, Part 2 Interest Rate

5.00% with probability 0.5

20.00% with probability 0.5

**Random Outcome:** %

When it is time to draw outcomes, you can click the “Draw Outcome” button whenever you feel ready. (You can try this now using the example scenarios, if you like.)

Important Note: The program generates new numbers very quickly (hundreds of times per second), so it is not possible for you to time your selection to choose any particular outcome.

When you finish drawing your outcomes, your Part 2 Payment for these scenarios will be computed, using the decisions you made and the outcomes you drew. Your total cash payment today will be equal to the sum of these Part 2 Payments.